1. [10 pt] Sequential search and binary search represent a tradeoff between search time and preprocessing time. How many binary searches need be performed in an \( n \)-element table to buy back the preprocessing time required to sort the table?

2. [10 pt] Give the asymptotic upper and lower bounds for \( T_n \) in the following recurrences. Assume that \( T_n \) is constant for \( n \leq 2 \). Make your bounds as tight as possible and justify your answers.

   (a) \( T_n = 7T_{n/3} + n^2 \). Use a method of your choice.

   (b) \( T_n = 7T_{n/2} + n^2 \). Use substitution method.
3. [10 pt] Consider a graph $G = (V, E)$. $G$ is undirected and connected. Consider the following statement about $G$: “If $G$ has a cycle with a unique heaviest edge $e$, then $e$ cannot be part of any minimum spanning tree.” Is this statement correct? If so, prove it. If it is not correct, give a counterexample.

4. [10 pt] Consider a tree with the set of vertices $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. The root is vertex 1. The edges and their weights are: $(1 - 2, 1), (1 - 3, 5), (2 - 4, 4), (3 - 5, 4), (3 - 6, 4), (4 - 7, 2), (4 - 8, 3), (6 - 9, 5), (6 - 10, 4)$. Solve the TVSP for this tree when
   
   (a) $\delta = 8$
   
   (b) $\delta = 4$
5. [10 pt] Consider a graph with $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$, and $E = \{(1 - 2, 4), (1 - 3, 8), (1 - 5, 1), (2 - 3, 3), (2 - 4, 5), (2 - 5, 3), (2 - 7, 5), (4 - 5, 6), (4 - 7, 9), (4 - 8, 2), (5 - 8, 6), (6 - 7, 10), (6 - 8, 3), (7 - 8, 2)\}$. Compute a minimum spanning tree for this graph using

(a) Prim’s algorithm

(b) Kruskal’s algorithm