**Important:** This is an open book test. You can use any books, notes, or paper. *Turn off your cell phone or any other electronic communication device.* Any calculations and rough work can be done on the back side of the test pages. You will lose five points for not writing your name.

1. [10 pt] Show that $6n^3 \neq \Theta(n^2)$.

2. [10 pt] Show by induction that $\sum_{i=0}^{n} i^2 = \Theta(n^3)$. 
3. [15 pt] Consider the following adjacency matrix representation of an undirected graph. The graph is presented as an upper triangular matrix as the matrix will be symmetric across the principal diagonal. The intersection of row $i$ and column $j$ gives the weight of the edge $ij$.

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & - & 11 & 13 & 2 & & & \\
2 & - & 15 & 8 & 12 & 6 & & \\
3 & - & & & & & & \\
4 & - & 14 & 10 & 17 & & & \\
5 & - & & & 5 & & & \\
6 & - & 21 & 7 & & & & \\
7 & - & & 11 & & & & \\
8 & - & & & & & & \\
\end{array}
\]

Draw this graph and compute its minimum cost spanning tree by Prim’s algorithm and Kruskal’s algorithm.
4. [10 pt] The worst case time for the merge sort algorithm is $O(n \lg n)$. What is its best case? Can we say that the time for merge sort is $\Theta(n \lg n)$?

5. [10 pt] Give an algorithm to compute the number of leaf nodes in a binary tree $t$. What is its computing time?
6. [4+6 pt] Consider the following adjacency matrix representation of a directed graph.

\[
\begin{matrix}
0 & 53 & 49 & 78 \\
50 & 0 & 64 & 69 \\
95 & 46 & 0 & 12 \\
27 & 33 & 15 & 0
\end{matrix}
\]

Solve the traveling salesperson problem on this graph using greedy method and dynamic programming. Show all work.