1. [25 pt] Show that if an undirected graph with \( n \) vertices has \( k \) connected components, then it has at least \( n - k \) edges.

2. [25 pt] Let \( T \) be a minimum spanning tree of graph \( G \). Given a connected subgraph \( H \) of \( G \), show that \( T \cap H \) is contained in some minimum spanning tree of \( H \).

3. [25 pt] Show that if \( G \) is a connected undirected graph, then no edge of \( G \) can be in two different biconnected components.

4. [50 pt] Write two programs to compute a minimum spanning tree for a given graph using Prim’s and Kruskal’s algorithms. The input will be given as follows:

\[
\begin{align*}
n \\
a & b & w_{ab} \\
c & d & w_{cd} \\
\end{align*}
\]

where \( n \) is the number of nodes, and \( a, b, c, d \) are integers in the range \([0, n)\). The last number on each line shows the weight for that edge.

**What to handin**

Hand in a hardcopy of all the sources, readme, makefile(s), and results. Create your programs in a directory called \$HOME/$USER.3 on admiral. Once you are done with everything, remove the executables and object files, and issue the following commands:

```
% cd
% ~bhatias/bin/handin cs5130 3
```