

LR(1) Item Sets

Definitions

An “LR(1) item” is a production with a mark on the right hand side (I use the symbol “ $\hat{}$ ” for the mark), together with a terminal.

A “complete item” is an item in which the mark is at the far right hand side of the item.

The set of viable prefixes for a context free language form a regular set; hence, there is a finite state machine that recognizes them. The transition symbols are language symbols (terminal and non-terminal); the states are identified by items or sets of items. There are two ways to produce it:

(I assume that the start symbol appears on the left of only one production; to make that happen, we could introduce a new start symbol S' and a production $S' \rightarrow S$, where S is the old start symbol.)

non-deterministic: (the states are items)

1. The start state is the state $[S' \rightarrow \hat{S}, -]$.
2. For any state $[A \rightarrow \alpha \hat{B} \beta, a]$, B a non-terminal, α and β strings of terminals and non-terminals (possibly empty), “ a ” a terminal, we have a transition on ϵ to all states $[B \rightarrow \hat{\gamma}, b]$, γ a string of language symbols, “ b ” an element of $\text{FIRST}(\beta a)$.
3. For any state $[A \rightarrow \alpha \hat{\Gamma} \beta, a]$, α and β strings of terminals and non-terminals, Γ a single symbol (terminal or non-terminal), “ a ” a terminal, we have a transition on Γ to the state $[A \rightarrow \alpha \Gamma \hat{\beta}, a]$.

Then we can transform into the deterministic FSM.

deterministic (directly): (the states are sets of items)

1. **Start Rule** The start state contains the item $[S' \rightarrow \hat{S}, -]$.
2. **Competition Rule** If a state contains an item $[A \rightarrow \alpha \hat{B} \beta, a]$, B a non-terminal, α and β strings of terminals and non-terminals (possibly empty), “ a ” a terminal, then the state also contains all items of the form $[B \rightarrow \hat{\gamma}, b]$, γ a string of language symbols, b an element of $\text{FIRST}(\beta a)$.
3. **Read Rule** If a state contains a subset of items $[A_i \rightarrow \alpha_i \hat{\Gamma} \beta_i, a_i]$, Γ a language symbol (the same for all such states), α_i and β_i strings of terminals and non-terminals (possibly empty), a_i terminals, then there is a transition of Γ to a state containing all $[A_i \rightarrow \alpha_i \Gamma \hat{\beta}_i, a_i]$ (the mark moves one position).

If the deterministic machine built as above for the language from its grammar has the property that all the Shift/Reduce and Reduce/Reduce conflicts are resolved, then the grammar is said to be an LR(1) grammar.