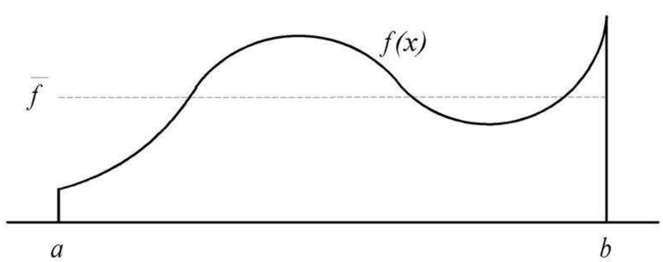
Monte Carlo Methods

Introduction

- Solve a problem using statistical sampling
  - First important use in development of atomic bomb during WWII
- Appliactions of Monte Carlo Methods
  - Evaluating integrals of arbitrary functions of 6+ dimensions
  - Predicting future value of stocks
  - Solving partial differential equations
  - Sharpening satellite images
  - Modeling cell populations
  - Finding approximate solutions to NP-hard problems in polynomial time
  - We used it as an exercise to compute the value of π
- Absolute error
  - A way to measure the quality of an estimate
  - The smaller the error, the better the estimate
  - Actual value denoted by a
  - Estimated (computed) value denoted by e
  - Absolute error $= \frac{|e-a|}{a}$
  - In the case of π, the absolute error is closely appromiated by the function $\frac{1}{2\sqrt{n}}$
  - Increasing sample size reduces error
- Why Monte Carlo works
  - Mean value theorem

    $$I = \int_a^b f(x)dx = (b-a)\bar{f}$$

    where $\bar{f}$ is the mean value of $f(x)$ in the intervals $[a, b]$



  - Monte Carlo method estimates the value of I by evaluating $f(x_i)$ at n points selected from a uniform random distribution over $[a, b]$
  - Expected value of $\frac{1}{n}\sum_{i=0}^{n-1} f(x_i)$ is $\bar{f}$

  - Hence, we have

$$
\begin{aligned}
I &= \int_a^b f(x)dx \\
&= (b-a)\bar{f} \\
&\approx (b-a)\frac{1}{n}\sum_{i=0}^{n-1} f(x_i)
\end{aligned}
$$

- Why Monte Carlo is effective
  - Error in Monte Carlo estimate decreases by the factor $\frac{1}{\sqrt{n}}$
  - Rate of convergence independent of integrand's dimension
  - Deterministic numerical integration methods do not share this property
    * Simpson's rule has rate of convergence that decreases as the dimension increases
  - Hence, Monte Carlo method is superior when integrand has 6 or more dimensions
- Monte Carlo and parallel computing
  - Monte Carlo methods often amenable to parallelism
  - Negligible amount of interprocess communications