

# Color Image Processing

## Background

- Humans can perceive thousands of colors, and only about a couple of dozen gray shades (cones/rods)
- Divide into two major areas: full color and pseudo color processing
  - Full color – Image is acquired with a full-color sensor like TV camera or color scanner
  - Pseudo color – Assign a color to a range of monochrome intensities
  - The availability of inexpensive and powerful hardware has resulted in the proliferation of applications based on full color processing
- 8-bit color vs 24-bit color
  - Color quantization
- Some of the gray scale image processing methods are directly applicable to color processing but others will need reformulation

## Color fundamentals

- Color spectrum/prism
  - Figure 7.1
  - White light divided into different colors
  - Colors blend into each other smoothly (Figure 7.2)
- Color – Perceptual result of light in the visible region of spectrum as incident on the retina
  - 400 nm to 700 nm
  - Visible light is a narrow band of frequencies in the electromagnetic spectrum (Figure 6.2)
  - White light is result of reflected light balanced across all visible wavelengths
  - Reflectance from a body in limited range of visible spectrum is perceived as color
    - \* Green objects reflect light with wavelength in the 500-570nm range while absorbing other wavelengths
- Characterization of light
  - Achromatic (no color) or monochromatic light characterized by intensity
  - Gray level as a scalar measure of intensity from black to white
- Chromatic light
  - Spans the electromagnetic spectrum from approximately 400–700nm
  - Light source characterized by three quantities
    - Radiance** Total amount of energy emitted by light source
      - \* Physical power of light energy, measured in watts
      - \* Directional quantity
      - \* Measures the quantity of radiation that passes through or emitted from a surface and falls within a given solid angle in a specified direction
      - \* Expressed in a spectral power distribution, often in 31 components, each representing a 10 nm band
      - \* Historically, also called *intensity*

**Brightness** Achromatic notion of intensity to describe color sensation

- \* Attribute of a visual sensation according to which an area appears to emit more or less light
- \* Subjective attribute of an object being observed
- \* Cannot be measured quantitatively

**Luminance** Measure of amount of energy as *perceived* by an observer, measured in lumens or candelas per square meter

- \* Light may contain a lot of energy in IR bands but that is not perceptible to the observer
  - \* More tractable version of brightness, defined by CIE
  - \* Radiant power weighted by a spectral sensitivity function that is characteristic of vision
  - \* Luminous efficiency peaks at 555nm
  - \* CIE luminance, denoted by  $Y$ , is the integral of spectral power distribution, using spectral sensitivity curve as a weighting function
  - \* Magnitude of luminance is proportional to physical power, but spectral composition is related to brightness sensitivity of human vision
  - \* Units of measurement for image processing
    - Normalized to 1 or 100 with respect to a standard white reference
    - $Y = 1$  is the white reference of a studio broadcast monitor whose luminance is 80 cd/m<sup>2</sup>
- Cones in the eye respond to three colors: red, green, blue
- \* 6 to 7 million cones in human eye
  - \* 65% cones respond to red eye
  - \* 33% cones respond to green light
  - \* 2% cones respond to blue light, these being most sensitive
  - \* Figure 7.3
  - \* Red, green, and blue are known as primary colors
    - In 1931, CIE designated specific wavelengths for primary colors
    - Red – 700nm
    - Green – 546.1nm
    - Blue – 435.8nm
    - To generate all colors, we may have to vary the wavelengths of primary colors while mixing colors; so the three primary colors are neither fixed nor standard
    - The curves in Figure 7.3 indicate that a single color may be called red, green, or blue
- Secondary colors
- \* Created by adding primary colors
  - \* Cyan = Green + Blue
  - \* Magenta = Red + Blue
  - \* Yellow = Red + Green
  - \* Mixing all three primary colors produces white
  - \* Figure 7.4
  - \* The secondary colors are primary colors of pigments, which have red, green, and blue as secondary colors
- How do we represent black? Absence of color.
- \* While printing, we need to print black on white
  - \* Subtractive colors based on pigments
  - \* Primary color of a pigment is defined as one that absorbs a primary color of light and transmits the other two
  - \* Given by cyan, magenta, yellow (CMY)
  - \* A secondary combined with its opposite primary produces black
- Color TV reception
- \* Characterized by additive nature of colors

- \* Large array of triangular dot patterns of electron sensitive phosphor
- \* Intensity of individual phosphors modulated by electron gun, one corresponding to each primary color
- \* The same technology is used in the flat panel displays, using three subpixels to generate a color pixel
- \* LCDs use properties of polarized light to block/pass light through LCD screen
  - Active matrix display technology uses thin film transistors to provide proper signal to each pixel on screen
- Color characterized by three quantities
 

**Hue** Dominant color as perceived by an observer (red, orange, or yellow)

**Saturation** Relative purity of color; pure spectrum colors are fully saturated

  - \* Saturation is inversely proportional to the amount of white light added

**Brightness** Achromatic notion of intensity
- Chromaticity
  - \* Combination of hue and saturation
  - \* Allows a color to be expressed as its brightness and chromaticity
- Tristimulus values
  - \* Three types of cones in the eye require three components for each color, using appropriate spectral weighting functions
    - Based on standard curves/functions defined by CIE – Commission Internationale de L'Éclairage
    - Curves specify the transformation of spectral power distribution for each color into three numbers
  - \* Amount of red, green, and blue to express a color
  - \* Denoted by  $X$ ,  $Y$ , and  $Z$
  - \* Color specified by its tristimulus coefficients

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

- \* Note that  $x + y + z = 1$
- \* Any wavelength of light in visible spectrum may be expressed by its tristimulus values from curves or tables compiled from experimental results
- Chromaticity diagram
  - \* Figure 7.5
  - \* Color given as a function of  $x$  and  $y$
  - \* The corresponding value of  $z$  is obtained by  $1 - (x + y)$
  - \* Points on the boundary are fully saturated colors
  - \* Saturation at point of equal energy is 0
  - \* Mainly useful for color mixing
    - Any straight line joining two points defines all the color variations obtained by combining the two colors additively
    - Extension to three colors by using a triangle to connect three points
    - Supports the assertion that not all colors can be obtained with three single, fixed primaries as some of them are outside the triangle
    - Figure 7.6 – Color gamut
 

Triangle encloses colors produced by RGB monitors

Shaded region inside the gamut shows the colors printed by high quality printers

## Color models

- Also called color space or color system
- Allow the specification of colors in some standard way
- Specification of a coordinate system and a subspace within that system
  - Each color represented by a single point
- Models oriented towards hardware (rendering and scanning) or software (reasoning and applications)
  - RGB for monitors/cameras
  - CMY and CMYK for printing
  - HSI for human-like reasoning and interpretation
- RGB color model
  - Figure 7.7
  - Unit cube
    - \* Based on Cartesian coordinate system
    - \* All color values are assumed to be normalized to the range [0,1]
    - \* Colors defined by vectors extending from origin; origin represents black
    - \* RGB primaries are at the corners that are neighbors to the origin; other corners (at distance 2 from origin) represent secondary colors (CMY)
    - \* Corner opposite to origin, given by point (1,1,1), represents white
    - \* Different shades of gray are distributed along the cube diagonal from black to white corners
  - Pixel depth – Number of bits used to represent each pixel in RGB space
    - \* Depth of 24-bits when each color represented by 8 bits in the triplet to represent pixel
    - \* Figure 6-08
  - Rendering an image
    - \* Images consist of three component images, one for each primary color
    - \* Figure 6-09
    - \* Fuse the three color components together
  - Acquiring an image
    - \* Figure 6-09, but in reverse
    - \* Acquire individual color planes and put them together
  - Does not make sense to use all the possible  $2^{24}$  colors in 24-bit space
    - \* Safe colors
      - Can be reproduced on a variety of devices
      - Likely to be reproduced faithfully, reasonably independent of hardware capabilities
    - \* Safe RGB colors or safe browser colors
      - Number of colors that can be reproduced faithfully in any system – 256
      - 40 of these colors are known to be processed differently by different OSs
      - Number of colors common to most systems – 216
    - \* Safe RGB color values
      - Formed from 6 possible values of each component as follows

Number System	Color Equivalents					
Hex	00	33	66	99	CC	FF
Decimal	00	51	102	153	204	255

- Each successive color is 51 (0x33) more than its predecessor
- Triplets give  $6^3 = 216$  possible values
- Figure 6-10
- Not all possible 8-bit gray colors are included in the set of 216 colors
- RGB safe-color cube – Figure 6.11
- Color safe cube has valid colors only on the surface

- CMY and CMYK color models

- Primary colors of pigments
- Pigments subtract light rather than radiate light
  - \* Illuminating a surface coated with cyan pigment absorbs red component of light
- Devices that deposit color pigments on paper perform an RGB to CMY conversion internally by a simple operation

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- Equal contribution of cyan, magenta, and yellow should produce black but in practice, it produces muddy-looking black
  - \* This may be also due to the fact that the CMY inks may not be pure
  - \* A fourth color is added, yielding CMYK system
- Conversion from CMY to CMYK

$$K = \min(C, M, Y)$$

- \*  $K = 1$  implies no color contribution, or

$$\begin{aligned} C &= 0 \\ M &= 0 \\ Y &= 0 \end{aligned}$$

- \* If  $K \neq 1$

$$\begin{aligned} C &= (C - K)/(1 - K) \\ M &= (M - K)/(1 - K) \\ Y &= (Y - K)/(1 - K) \end{aligned}$$

- Conversion from CMYK back to CMY

$$\begin{aligned} C &= C \times (1 - K) + K \\ M &= M \times (1 - K) + K \\ Y &= Y \times (1 - K) + K \end{aligned}$$

- Indexed or palette image

- Uses a fixed number of colors within the color or gray component of an image
- Image values are just indices in a table of color values

- HSI color model

- Hue, saturation, intensity
- RGB and CMY models
  - \* Ideally suited for hardware implementation

- \* RGB matches the human eye's perception for primary colors
- \* RGB and CMY not suitable for describing colors for human interpretation
- \* Dark or light or pastel colors
- \* Humans do not think of color images as being composed of three primary images that form a single image
- Human description of images/colors
  - \* In terms of hue, saturation, and brightness
- HSI model decouples intensity component from the color-carrying components (hue and saturation)
  - \* Ideal tool for developing image processing algorithms
  - \* Natural and intuitive to humans
- Intensity
  - \* Measure over some interval of the electromagnetic spectrum of the flow of power that is radiated from, or incident on, a surface
  - \* Linear light measure, expressed in units such as watts per square meter
  - \* Controlled on a CRT monitor by voltages presented, in a nonlinear manner for each color component
  - \* CRT voltages are not proportional to intensity
  - \* RGB color images can be viewed as three monochrome intensity images
  - \* Extracting intensity from RGB images
    - Stand the RGB color cube on the black vertex, with white vertex directly above it (Figure 7.10a)
    - Line joining the black and white vertices is now vertical
    - Intensity of any color given by intersection of intensity axis and a plane perpendicular to it and intersecting with the color point in cube
    - Saturation of color increases as a function of distance from intensity axis
    - Saturation of points along intensity axis is zero (all points on intensity axis are gray)
- Hue
  - \* Color attribute that describes a pure color
  - \* Consider the plane defined by black, white, and cyan (Figure 7.10b)
  - \* Intensity axis is contained within this plane
  - \* All points contained in plane segment given by these three points have the same hue – cyan
  - \* Rotating the plane about the intensity axis gives us different hues
  - \* HSI space is represented by a vertical intensity axis and the locus of color points that lie on planes perpendicular to the axis
    - As planes move up and down on intensity axis, the boundaries of intersection of each plane with the faces of the cube have a triangular or hexagonal shape
- Above discussion leads us to conclude that we can convert a color from the RGB values to HSI space by working out the geometrical formulas (Figure 7.11)
  - \* Primary colors are separated by  $120^\circ$
  - \* Secondary colors are  $60^\circ$  from the primaries
  - \* Hue of a point is determined by an angle from a reference point
    - By convention, reference point is taken as angle from red axis
    - Hue increases counterclockwise from red axis
  - \* Saturation is the length of vector from origin to the point
    - Origin is given by intensity axis
- Figure 7.12 to describe HSI model
- Converting colors from RGB to HSI
  - Consider RGB values normalized to the range  $[0, 1]$

- Given an RGB value,  $H$  is obtained as follows:

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases}$$

\* It should be normalized to the range  $[0, 1]$  by dividing the quantity computed above by 360

- $\theta$  is given by

$$\theta = \cos^{-1} \left\{ \frac{\frac{1}{2}[(R - G) + (R - B)]}{[(R - G)^2 + (R - B)(G - B)]^{1/2}} \right\}$$

\*  $\theta$  is measured with respect to red axis of HSI space

- Saturation is given by

$$S = 1 - \frac{3}{(R + G + B)}[\min(R, G, B)]$$

- Intensity component is given by

$$I = \frac{1}{3}(R + G + B)$$

- Converting colors from HSI to RGB

- Consider the values of HSI in the interval  $[0, 1]$
- $H$  should be multiplied by 360 (or  $2\pi$ ) to recover the angle; further computation is based on the value of  $H$
- RG sector –  $0^\circ \leq H < 120^\circ$

$$\begin{aligned} B &= I(1 - S) \\ R &= I \left[ 1 + \frac{S \cos H}{\cos(60^\circ - H)} \right] \\ G &= 3I - (R + B) \end{aligned}$$

- GB sector –  $120^\circ \leq H < 240^\circ$

$$\begin{aligned} H' &= H - 120^\circ \\ R &= I(1 - S) \\ G &= I \left[ 1 + \frac{S \cos H'}{\cos(60^\circ - H')} \right] \\ B &= 3I - (R + G) \end{aligned}$$

- BR sector –  $0^\circ \leq H < 360^\circ$

$$\begin{aligned} H' &= H - 240^\circ \\ G &= I(1 - S) \\ B &= I \left[ 1 + \frac{S \cos H'}{\cos(60^\circ - H')} \right] \\ R &= 3I - (G + B) \end{aligned}$$

- Figure 7.13

- HSI components of RGB cube, plotted separately
- Discontinuity along the  $45^\circ$  line in the hue figure
  - \* See the reason by going around the middle in Figure 7.8
- Saturation image shows progressively darker values close to the white vertex of RGB cube
- Intensity is simply the average of RGB values at the corresponding pixel

- Manipulating HSI component images
  - Figure 7.14 – image composed of primary and secondary RGB colors and their HSI equivalents
    - \* In hue, red region maps to black as its angle is  $0^\circ$
    - \* In b, c, and d parts of the image, the pixels are scaled to the range  $[0,1]$
  - Individual colors changed by changing the hue image
  - Purity of colors changed by varying the saturation
  - Figure 7.15a – Change blue and green pixels in Figure 7.14a to 0 (compare with Figure 7.14b)
  - Figure 7.15b – Change saturation of cyan component in Figure 7.14c to half
  - Figure 7.15c – Reduce the intensity of central white region in Figure 7.14d by half
  - Figure 7.15d – Combine the three HSI components back into RGB image

### HSV color space

- Projects the RGB color cube onto a non-linear chroma angle (H), a radial saturation percentage (S), and a luminance-inspired value (V)
- Similar to HSI color space
- Used to compare the hue channel in OpenCV

### Pseudocolor image processing

- Also known as *indexed color* or *false color*
- Assign colors to gray values based on a fixed criteria
  - 216 index entries from 8-bit RGB color system as a  $6 \times 6 \times 6$  cube in a direct color system
  - Gives an integer in the range 0 to 5 for each component of RGB
  - Requires less data to encode an image
  - Some graphics file formats, such as GIF and TIFF add an index colormap to the image with gamma-corrected RGB entries
- Used as an aid to human visualization and interpretation of gray-scale events in an image or sequence of images, such as visualizing population density or temperature in different areas on a map

$$g(x, y) = \begin{cases} 0xFF0000 & \text{if } g(x, y) > 100 \\ 0xFF00FF & \text{if } 90 < g(x, y) \leq 100 \\ 0xFFFF00 & \text{if } 80 < g(x, y) \leq 90 \\ 0X00FF00 & \text{if } 70 < g(x, y) \leq 80 \\ 0X00FFFF & \text{if } 60 < g(x, y) \leq 70 \\ 0X0000FF & \text{otherwise} \end{cases}$$

- May have nothing to do with processing of true color images
- Intensity slicing
  - Also called density slicing or color coding
  - Slicing planes parallel to horizontal plane in 3D space, with the intensity of image providing the third dimension on image plane
    - \* Figure 7.16



- \* Plane at  $f(x, y) = l_i$  to slice the image function into two levels
- \* Assign different colors to area on different sides of the slicing plane
- \* Relative appearance of the resulting image manipulated by moving the slicing plane up and down the gray-level axis
- Technique summary
  - \* Gray scale representation –  $[0, L - 1]$
  - \* Black represented by  $l_0$ ,  $[f(x, y) = 0]$
  - \* White represented by  $[l_{L-1}]$ ,  $[f(x, y) = L - 1]$
  - \* Define  $P$  planes perpendicular to intensity axis at levels  $l_1, l_2, \dots, l_P$
  - \*  $0 < P < L - 1$
  - \*  $P$  planes partition the gray scale into  $P + 1$  intervals as  $I_1, I_2, \dots, I_{P+1}$
  - \* Make gray-level to color assignment as

$$f(x, y) = c_k \quad \text{if } f(x, y) \in I_k$$

where  $c_k$  is the color associated with  $k$ th intensity interval  $I_k$  defined by partitioning planes at  $l = k - 1$  and  $l = k$

- Alternative mapping function to intensity slicing planes
  - \* Figure 7.17
  - \* Staircase form of mapping with multiple levels
- Figure 7.18 – Picker Thyroid Phantom (radiation test pattern)
  - \* Intensity slicing image into eight color regions
  - \* Idea is to make it easy to distinguish between shades without assigning any semantic interpretation to the color
  - \* Characteristics of intensity variations in grayscale image are more apparent by varying the number of colors and the span of intensity intervals
- Figure 7.19
  - \* Full strength of X-rays passing through is assigned one color; everything else a different color
- Figure 7.20 – Measurement of rainfall levels
- [Current temperature map of US](#)
- Gray level to color transformations
  - Separate independent transformation of gray level inputs to three colors
  - Figure 7.21
  - Composite image with color content modulated by nature of transformation function
  - Piecewise linear functions of gray levels
  - Figure 7.22 – Luggage through X-ray scanning system
    - \* Image on right contains a block of simulated plastic explosives
  - \* Figure 7.23 – Transformation functions used
  - \* Emphasize ranges in gray scale by changing sinusoidal frequencies
- Combining several monochrome images into a single color composite
  - Figure 7.24
  - Used in multispectral image processing, with different sensors producing individual monochrome images in different spectral bands
  - Figure 7.25
    - \* Images of Washington, DC, and Potomac river in red, green, blue, and near IR bands

- \* Image  $f$  generated by replacing the red component of image  $e$  by NIR image
  - NIR strongly responsive to biomass component
- \* Image  $f$  shows the difference between biomass (red) and man-made features such as concrete and asphalt (bluish green)
- Figure 7.26
  - \* Jupiter moon Io, using images in several spectral regions by the spacecraft Galileo
  - \* Bright red depicts material recently ejected from an active volcano while surrounding yellow shows older sulfur deposits
- Chroma key compositing or Chroma keying
  - Used in movies and TV broadcasting to separate foreground from background to blend multiple images
  - Make a color range in the foreground region transparent, and insert separately filmed background footage into the scene
  - Most common use is weather forecasting using green screen
    - \* Green (or blue) color used more often because they are distinct from human skin tones
    - \* The camera is most sensitive to the green light
    - \* Notice that the weatherman will never appear in a green tie or any other green-colored dress
  - Produce a mask by thresholding the amount of green color

$$M(x, y) = \begin{cases} 1 & \text{if } I_G(x, y) > \tau \\ 0 & \text{otherwise} \end{cases}$$

- [Example of memes with Melania Trump during Republican National Convention](#)

## Basics of full-color image processing

- Two major categories of processing
    1. Process each component of image (RGB or HSI) individually and then form a composite processed color image
      - Each component can be processed using gray-scale processing techniques
    2. Work with color pixels directly, treating each pixel as a vector
- $$\text{Color vector } \mathbf{c} = \begin{bmatrix} c_R \\ c_G \\ c_B \end{bmatrix} = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$
- Since each pixel is a function of coordinates  $(x, y)$ , we have
 
$$\mathbf{c}(x, y) = \begin{bmatrix} c_R(x, y) \\ c_G(x, y) \\ c_B(x, y) \end{bmatrix} = \begin{bmatrix} R(x, y) \\ G(x, y) \\ B(x, y) \end{bmatrix}$$
    - Each component of the vector is a *spatial* variable in  $x$  and  $y$
    - For an  $M \times N$  image, there are  $MN$  vectors  $\mathbf{c}(x, y)$  for  $x = 0, 1, 2, \dots, M - 1$  and  $y = 0, 1, 2, \dots, N - 1$
    - As the data in a pixel contains two or more components, we need a third dimension to represent the pixel, leading to the term *voxel*, or *volumetric pixel*
- The two methods may or may not produce equivalent results
    - Scalar versus vector operations
      - \* The process used should be applicable to both scalars and vectors

- \* The operation on each component of the vector must be independent of the other components
- Neighborhood processing may be an example where we get similar or different results (Figure 7.27)
- \* Averaging the images separately in individual planes and averaging the vectors will give the same result

### Color transformations

- Process the components of a color image within the context of a single color model, without converting components to different color space
- Think of an application that needs to brighten a picture
  - Can we achieve this by adding a constant quantity to each of the three RGB channels?
  - This will not only increase the intensity of each pixel but also hue and saturation
  - A better solution will be to manipulate the luminance  $I$  to recompute a valid RGB image with the same hue and saturation
- Formulation
  - Model color transformations using the expression

$$g_i(x, y) = T_i[f_i(x, y)] \quad i = 1, 2, \dots, n$$

$n$  is the number of component images or channels,  $T_i$  is the set of transformation (or color mapping functions) over a spatial neighborhood of  $(x, y)$

- For RGB images, each  $f(x, y)$  component is a triplet in the chosen color space (Figure 7.27)
- The subscript  $i$  for  $T$  indicates that each component may have a different transform
- Figure 7.28 – Various color components of an image
  - \* The transform  $T_i$  may be applied to any component of the image
- Must consider the cost of converting from one color space to another when looking at the operations
- Modifying intensity of an image in different color spaces, using the transform

$$g(x, y) = kf(x, y)$$

- \* In HSI color space, converting a pixel  $h, s, i$  to  $h', s', i'$

$$\begin{aligned} h' &= h \\ s' &= s \\ i' &= ki \end{aligned}$$

- \* In RGB color space, converting a pixel  $r, g, b$  to  $r', g', b'$

$$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = k \cdot \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

- \* In CMY color space

$$\begin{aligned} c' &= kc + (1 - k) \\ m' &= km + (1 - k) \\ y' &= ky + (1 - k) \end{aligned}$$

- \* In CMYK space

$$g_i(x, y) = \begin{cases} f_i(x, y) & i = 1, 2, 3 \\ kf_i(x, y) + (1 - k) & i = 4 \end{cases}$$

- Simple operation in HSI but cost to convert to HSI may not be justifiable
  - \* Figure 7.29, using  $k = 0.7$
- Application of transform to each component is independent of other components
- Color complements
  - Hues directly opposite one another on the color circle
    - \* Figure 7.30
  - Analogous to gray scale negatives
  - Can be used to enhance details buried in dark regions of an image
  - Figure 7.31
    - \* May not have the same saturation in negative image in HSI
    - \* Figure shows saturation component unaltered
- Color slicing
  - Used to highlight a specific range of colors in an image to separate objects from surroundings
  - Display just the colors of interest, or use the regions defined by specified colors for further processing
  - More complex than gray-level slicing, due to multiple dimensions for each pixel
    - \* Practical color-slicing approaches require each pixel's transformed components to be a function of all components of the original pixel
  - Dependent on the color space chosen; I prefer HSI
  - Mapping unwanted colors (outside of specified range) to a neutral color
    - \* Use a cube (or hypercube) of width  $W$  to enclose the reference color with components  $(a_1, a_2, \dots, a_n)$
    - \* Transformation is given by
 
$$s_i = \begin{cases} 0.5 & \text{if } [|r_j - a_j| > \frac{W}{2}]_{\text{any } 1 \leq j \leq n} \\ r_i & \text{otherwise} \end{cases} \quad i = 1, 2, \dots, n$$
    - \* If the color of interest is specified by a sphere of radius  $R_0$ , the transformation is
 
$$s_i = \begin{cases} 0.5 & \text{if } \sum_{j=1}^n (r_j - a_j)^2 > R_0^2 \\ r_i & \text{otherwise} \end{cases} \quad i = 1, 2, \dots, n$$
    - \* Figure 7.32 – Separate the strawberries in Figure 7.29a
      - Select RGB color (0.6863, 0.1608, 0.1922)
      - $W = 0.2549$ ,  $R = 0.1765$
- Color balancing
  - Process to compensate for incandescent lighting
  - You can perform color balancing by multiplying each channel with a different scale factor, or by mapping the pixels to XYZ color space, changing the nominal white point, and mapping back to RGB

## Tone and color corrections

- Used for photo enhancement and color reproduction
  - Tonal range of an image
    - \* Also called key-type
    - \* Gives general distribution of color intensities

- \* High key, middle key, and low key images
- \* Distribute intensities equally between highlights and shadows
- Device independent color model from CIE relating the color gamuts
- Use a color profile to map each device to color model
- CIE L\*a\*b\* system
  - Most common model for color management systems
  - Components given by the following equations

$$\begin{aligned}
 L^* &= 116 \cdot h\left(\frac{Y}{Y_W}\right) - 16 \\
 a^* &= 500 \left[ h\left(\frac{X}{X_W}\right) - h\left(\frac{Y}{Y_W}\right) \right] \\
 b^* &= 200 \left[ h\left(\frac{Y}{Y_W}\right) - h\left(\frac{Z}{Z_W}\right) \right]
 \end{aligned}$$

where

$$h(q) = \begin{cases} q^{\frac{1}{3}} & \text{if } q > 0.008856 \\ 7.787q + \frac{16}{116} & \text{otherwise} \end{cases}$$

- $X_W, Y_W$ , and  $Z_W$  are values for reference white, called  $D_{65}$  which is defined by  $x = 0.3127$  and  $y = 0.3290$  in the CIE chromaticity diagram
- $X, Y, Z$  are computed from RGB values as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

- \* Rec. 709 RGB corresponds to  $D_{65}$  white point
- L\*a\*b\* is calorimetric (colors perceived as matching are encoded identically), perceptually uniform (color differences among various hues are perceived uniformly), and device independent
- Not directly displayable on any device but its gamut covers the entire visible spectrum
- L\*a\*b\* decouples intensity from color, making it useful for image manipulation (hue and contrast editing) and image compression applications
  - \* L\* represents lightness or intensity
  - \* a\* gives red minus green
  - \* b\* gives green minus blue
- Allows tonal and color imbalances to be corrected interactively and independently
  - \* Tonal range refers to general distribution of key intensities in an image
    - Adjust image brightness and contrast to provide maximum detail over a range of intensities
    - The colors themselves are not changed
- Figure 7.33
  - \* Typical RGB transformations to correct three common tonal imbalances – flat, light, and dark images
  - \* Keys in the images are visually evident; they may be computed using histograms of color components
- Color balancing
  - Objectively performed using a color spectrometer
  - Can also be assessed visually using skin tones (Figure 7.34)

- Adjusting color components
  - \* Every action affects the overall color balance of the image
  - \* Perception of a color is affected by surrounding colors
  - \* Use color wheel (Figure 7.30) to increase the proportion of a color by decreasing the amount of complementary color
  - \* May also increase the proportion of a color by raising the contribution of its adjacent colors

## Histogram processing

- Provides an automated way to perform enhancement
- Histogram equalization
  - Adapt the grayscale technique to multiple components
  - Applying grayscale techniques to different colors independently yields erroneous colors
  - Spread the intensities uniformly leaving the hues unchanged
  - Figure 7.35 – histogram equalization followed by saturation adjustment in HSI color space

## Compositing

- Used for special effects in movies; blend live action with graphics
- Method to blend two layers into one another
- Dissolving
  - Simplest effect
  - Given two images  $I_1$  and  $I_2$  of the same size
  - Their weighted combination is given by

$$I'(x, y) = w_1 I_1(x, y) + w_2 I_2(x, y)$$

- $w_1$  and  $w_2$  are scalar weights with the condition that  $w_1 + w_2 = 1$
- Changing  $w_1$  slowly from 1 to 0, with  $w_2$  computed as  $1 - w_1$  slowly dissolves the first image into the second
- Compositing with binary masks
  - Let the two images be accompanied by binary masks  $M_1$  and  $M_2$  to define the support of pixels
    - \*  $M(x, y) = 1$  if  $I(x, y)$  is valid
    - \*  $M(x, y) = 0$  if  $I(x, y)$  is invalid
    - \* The main idea is to work just with the valid pixels
  - Binary masks allows for four cases for any given pixel in two images
    - $M_1(x, y) = M_2(x, y) = 0 \Rightarrow$  output pixel mask  $M'(x, y) = 0$ , or both inputs are invalid; value of input pixels is irrelevant
    - $M_1(x, y) = 0, M_2(x, y) = 1 \Rightarrow$  output pixel mask can be 0 (invalid) or 1 (valid); set output pixel to the only valid input  $I_2(x, y)$
    - $M_1(x, y) = 1, M_2(x, y) = 0 \Rightarrow$  output pixel mask can be 0 (invalid) or 1 (valid); set output pixel to the only valid input  $I_1(x, y)$
    - $M_1(x, y) = 1, M_2(x, y) = 1$  leads to three choices: output pixel can be selected from  $I_1(x, y)$  or  $I_2(x, y)$  or neither

- The binary masks give us 1 choice for the first, 2 choices for the second and third, and three choices for the fourth case

- \* This leads to  $1 \cdot 2 \cdot 2 \cdot 3 = 12$  possible ways to combine two images with binary masks
- \* The 12 compositing operators are known as *Porter-Duff operators*
  - With  $I_1$  and  $I_2$  as the input images and  $I'$  as the output image, the operators can be summarized as

Operator	Output $I'(x, y)$	Mask $M'(x, y)$
clear	0	0
copy $I_1$	$I_1(x, y)$	$M_1(x, y)$
copy $I_2$	$I_2(x, y)$	$M_2(x, y)$
$I_1$ over $I_2$	$(I_1 \wedge M_1) \vee (I_2 \wedge M_2 \wedge \neg M_1)$	$M_1 \vee M_2$
$I_1$ in $I_2$	$I_1$	$M_1 \wedge \neg M_2$
$I_1$ out $I_2$	$I_1$	$M_1 \wedge \neg M_2$

- *Blend Modes*

## Smoothing and sharpening

- Color image smoothing
  - Extend grayscale spatial filtering mask to color smoothing, dealing with component vectors
  - Let  $S_{xy}$  be the neighborhood centered at  $(x, y)$  in an RGB color image
  - Average of RGB components in the neighborhood is given by

$$\bar{\mathbf{c}}(x, y) = \frac{1}{K} \sum_{(s, t) \in S_{xy}} \mathbf{c}(s, t)$$

which is the same as

$$\bar{\mathbf{c}}(x, y) = \begin{bmatrix} \frac{1}{K} \sum_{(s, t) \in S_{xy}} R(s, t) \\ \frac{1}{K} \sum_{(s, t) \in S_{xy}} G(s, t) \\ \frac{1}{K} \sum_{(s, t) \in S_{xy}} B(s, t) \end{bmatrix}$$

- Same effect as smoothing each channel separately
- Figure 7.36 (RGB components), Figure 7.37 (HSI components)
  - \* Figure 7.38a – Smooth each component of RGB image independently
  - \* Figure 7.38b – Smooth only the intensity component of HSI
  - \* Pixel colors do not change as they do with RGB smoothing
  - \* Figure 7.38c – Difference between the two smoothed out images
  - \* More efficient to smooth in HSI; difference from RGB becomes more pronounced with increase in kernel size

- Color image sharpening

- Use Laplacian

$$\nabla^2[\mathbf{c}(x, y)] = \begin{bmatrix} \nabla^2 R(x, y) \\ \nabla^2 G(x, y) \\ \nabla^2 B(x, y) \end{bmatrix}$$

- Figure 7.39
  - \* a: Laplacian applied to RGB components independently
  - \* b: Laplacian applied to just the intensity component of HSI
  - \* c: Difference between the two sharpened images

## Image segmentation based on color

- Segmentation partitions images into regions
- Segmentation in HSI color space
  - Color is conveniently represented in hue component
  - Saturation is used as a masking image to isolate regions of interest in the hue component
  - Intensity image used less frequently as it has no color information
  - Example 7.14
    - \* Segment the reddish region in lower left of Figure 7.40a
    - \* Figure 7.40e: Binary mask by thresholding the saturation image with 10% of the maximum value in the image
    - \* Figure 7.40f: Product of hue and thresholded saturation
    - \* Figure 7.40g: Histogram of Figure 7.40f
- Segmentation in RGB vector space
  - Create an estimate of the average color to be segmented as vector  $\mathbf{a}$
  - Let  $\mathbf{z}$  be an arbitrary point in the RGB color space
  - $\mathbf{z}$  is similar to  $\mathbf{a}$  if the Euclidean distance between them is less than specified threshold  $D_0$

$$\begin{aligned}
 D(\mathbf{z}, \mathbf{a}) &= \|\mathbf{z} - \mathbf{a}\| \\
 &= [(\mathbf{z} - \mathbf{a})^T (\mathbf{z} - \mathbf{a})]^{\frac{1}{2}} \\
 &= [(\mathbf{z}_R - \mathbf{a}_R)^2 + (\mathbf{z}_G - \mathbf{a}_G)^2 + (\mathbf{z}_B - \mathbf{a}_B)^2]^{\frac{1}{2}}
 \end{aligned}$$

- Locus of points such that  $D(\mathbf{z}, \mathbf{a}) \leq D_0$  is a solid sphere of radius  $D_0$  (Figure 7.41a)
- Simple generalization provided by

$$D(\mathbf{z}, \mathbf{a}) = [(\mathbf{z} - \mathbf{a})^T \mathbf{C}^{-1} (\mathbf{z} - \mathbf{a})]^{\frac{1}{2}}$$

where  $\mathbf{C}$  is the covariance matrix of samples chosen to represent color range to be segmented

- \* Locus of points such that  $D(\mathbf{z}, \mathbf{a}) \leq D_0$  is a solid ellipsoid with principal axis oriented in the direction of maximum data spread (Figure 7.41a)
- \* When  $\mathbf{C} = \mathbf{I}$ , the ellipsoid reduces to sphere
- Distances are positive and monotonic
  - \* We can work with square of the distance to avoid computationally expensive square roots
  - \* Still, the computations are expensive for images of practical size
  - \* Compromise by using a bounding box (Figure 7.41c)
  - \* Box is centered on  $\mathbf{a}$ , with dimensions along the color axes proportional to the standard deviation of the samples along each axis
  - \* A given color point is segmented based on whether it is inside the box
- Example 7.15
  - \* Figure 7.42
  - \* Color to be segmented selected by rectangular region inside Figure 7.42a
  - \* Compute mean vector  $\mathbf{a}$  of the shades in the rectangle
  - \* Compute the standard deviation of the red, green, and blue components

## Color Edge Detection

- Computing edges in individual component planes in the image
  - Edge detection by gradient operator for image sharpening



- \* May not work with vector data for pixels
- \* Working on separate color channels and combining the result may not work either
- \* Figure 7.43
  - Results may be acceptable but not accurate
- Define the gradient (magnitude and direction) of the vector  $\mathbf{c}$  at any point  $(x, y)$ 
  - \* For a scalar function  $f(x, y)$ , gradient is a vector pointing in the direction of maximum rate of change of  $f$  at coordinates  $(x, y)$
  - \* Let  $\mathbf{r}$ ,  $\mathbf{g}$ , and  $\mathbf{b}$  be the unit vectors along the R, G, and B axes of RGB color space
  - \* Define the vectors

$$\mathbf{u} = \frac{\partial R}{\partial x} \mathbf{r} + \frac{\partial G}{\partial x} \mathbf{g} + \frac{\partial B}{\partial x} \mathbf{b}$$

$$\mathbf{v} = \frac{\partial R}{\partial y} \mathbf{r} + \frac{\partial G}{\partial y} \mathbf{g} + \frac{\partial B}{\partial y} \mathbf{b}$$

- \* The gradients are defined as the dot product of these vectors as

$$g_{xx} = \mathbf{u} \cdot \mathbf{u} = \mathbf{u}^T \mathbf{u} = \left| \frac{\partial R}{\partial x} \right|^2 + \left| \frac{\partial G}{\partial x} \right|^2 + \left| \frac{\partial B}{\partial x} \right|^2$$

$$g_{yy} = \mathbf{v} \cdot \mathbf{v} = \mathbf{v}^T \mathbf{v} = \left| \frac{\partial R}{\partial y} \right|^2 + \left| \frac{\partial G}{\partial y} \right|^2 + \left| \frac{\partial B}{\partial y} \right|^2$$

$$g_{xy} = \mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T \mathbf{v} = \frac{\partial R}{\partial x} \frac{\partial R}{\partial y} + \frac{\partial G}{\partial x} \frac{\partial G}{\partial y} + \frac{\partial B}{\partial x} \frac{\partial B}{\partial y}$$

- \* Direction of maximum rate of change of  $\mathbf{c}(x, y)$  is given by

$$\theta(x, y) = \frac{1}{2} \tan^{-1} \left[ \frac{2g_{xy}}{g_{xx} - g_{yy}} \right]$$

- \* Value of the rate of change at  $(x, y)$  in the direction of  $\theta(x, y)$  is given by

$$F_{\theta}(x, y) = \left\{ \frac{1}{2} [(g_{xx} + g_{yy}) + (g_{xx} - g_{yy}) \cos 2\theta(x, y) + 2g_{xy} \sin 2\theta(x, y)] \right\}^{\frac{1}{2}}$$

- \* Since  $\tan(\alpha) = \tan(\alpha \pm \pi)$ , if  $\theta_0$  is a solution to the equation for  $\theta$  above, so is  $\theta_0 \pm \pi/2$ 
  - Additionally,  $F_{\theta} = F_{\theta+\pi}$
  - The equation for  $\theta$  gives two values  $90^\circ$  apart, or a pair of orthogonal directions
  - $F$  is maximum along one direction and minimum along the other
- Example 7.16 – Edge detection in RGB vector space
  - \* Figure 7.44

## Image File Formats

- Files used to store, archive, and exchange image data
  - Standardized file formats facilitate the exchange of images and allow different applications to read those images
- Criteria to select appropriate file format
  - Image type
    - \* Binary, grayscale, or color images
    - \* Document scans, floating point images
    - \* Maximum image size for satellite images
  - Storage size and compression

- \* Lossy or lossless compression
- Compatibility
  - \* Exchange of image data with others and across applications
  - \* Long-term machine readability of data
- Application domain
  - \* Print, web, film, graphics, medicine, astronomy
- Raster vs vector data
  - All images considered thus far have been raster images
  - Vector graphics represent geometric objects using continuous coordinates
    - \* The objects are rasterized when they need to be displayed on a physical device
  - Used to encode geodata for navigation systems
- Tagged Image File Format (TIFF)
  - Supports grayscale, indexed, and true color images
  - A single file may contain a number of images with different properties
  - Provides a range of different compression methods (LZW, ZIP, CCITT, and JPEG), and color spaces
  - You can create new image types and information blocks by defining new *tags*
    - \* Proprietary tags may not be always supported leading to “unsupported tag” error
    - \* Web browsers do not natively support TIFF
- Graphics Interchange Format (GIF)
  - Originally designed by CompuServe in 1986
  - Provided early support for indexed color at various bit depths
  - Provided LZW compression, interlaced image loading, and ability to encode simple animations by storing a number of images in a single file for sequential display
  - Does not support true color images
  - Allows pixels to be encoded using fewer bits
  - Uses lossless color quantization and lossless LZW compression
- Portable Network Graphics (PNG)
  - Developed as a replacement for GIF because of licensing issues
  - Supports three different types of images
    1. True color, with up to  $3 \times 16$  bpp
    2. Grayscale, with up to 16 bpp
    3. Indexed, with up to 256 colors
  - Also may include an  $\alpha$ -channel for transparency with a maximum width of 16 bits
    - \*  $\alpha$ -channel of a GIF image is only 1 bit
  - Supports only one image per file, with maximum size as  $2^{30} \times 2^{30}$  pixels
    - \* Cannot support animation like GIF
  - Supports lossless compression by a variation of PKZIP but no lossy compression
- Joint Photographic Experts Group (JPEG)
  - Goal to achieve average data reduction of 1:16
  - Supports images with up to 256 color components

- Three steps in the core algorithm for RGB images
  1. Color conversion and down sampling
    - \* Transform from RGB to  $YC_bC_r$  space;  $Y$  is brightness while the other two components are color
    - \* Human visual system is less sensitive to rapid color change; compress color components more to achieve significant data reduction without a perceptive change in image quality
  2. Cosine transform and quantization in frequency space
    - \* Image is divided into a regular grid of  $8 \times 8$  blocks
    - \* Compute frequency spectrum of each block using discrete cosine transform
    - \* The 64 spectral components of each block are quantized into a quantization table
    - \* Reduce high frequency components and recompute them during decompression
  3. Lossless compression
    - \* Compress quantized spectral component data stream using arithmetic or Huffman encoding
- Not a good choice for images such as line drawings