

**Important:** This is an open book test. You can use any books, notes, or paper. *Do not log into the computer during the test.* Switch off any communications devices, including cell phones. Any calculations and rough work can be done on the back side of the test pages. You will lose five points for not writing your name.

1. [10 pt] Let  $T_n$  represent the time needed by pre-order traversal algorithm to traverse a binary tree when the input tree  $t$  has  $n \geq 0$  nodes. If the time needed to visit a node is  $\Theta(1)$ , prove that  $T_n = \Theta(n)$ .
2. [10 pt] Give a nondeterministic algorithm of complexity  $O(n)$  to determine whether there is a subset of  $n$  numbers  $a_i, 1 \leq i \leq n$ , that sums to  $m$ .

3. [15 pt] Let  $w = \{5, 7, 10, 12, 15, 18, 20\}$  and  $m = 35$ . Find all possible subsets of  $w$  that sum to  $m$ . Use a backtracking algorithm and draw the portion of the state space tree that is generated.

4. [10 pt] Draw the portion of the state space tree generated by LC branch-and-bound algorithm for the following 0/1-knapsack instance:

$$n = 5$$

$$P = (4, 4, 5, 8, 9)$$

$$W = (4, 4, 5, 8, 9)$$

$$m = 15$$

5. [10 pt] Consider the following instance of the 0/1-knapsack problem:

$$n = 5$$

$$P = (10, 15, 6, 8, 4)$$

$$W = (4, 6, 3, 4, 2)$$

$$m = 12$$

Give the optimal and approximate solutions for this problem. For your approximate instance, calculate the value of  $\rho(n)$ .