Important: This is an open book test. You can use any books, notes, or paper. Do not log into the computer during the test. Switch off any communications devices, including cell phones. Any calculations and rough work can be done on the back side of the test pages. You will lose five points for not writing your name.

1. [10 pt] Prove that if $G$ is a connected undirected graph with $n$ vertices and $n-1$ edges, then $G$ is a tree. Use the fact that a tree is a directed acyclic graph. The simplest proof seems to be by induction but you are welcome to use any other technique.

2. [10 pt] We used a backtracking approach to compute all possible subsets that sum to a given number. We used data that was already sorted in increasing order. Will it make a difference in our algorithm if the data is not sorted? Will we get all the subsets still? How is the performance affected.
3. [10 pt] Consider the following parameters for the 0/1 knapsack problem. $n = 5$, $P = (10, 15, 6, 8, 4)$, $W = (4, 6, 3, 4, 2)$, and $m = 12$. Use branch-and-bound algorithm to find an optimal solution for this problem. Do not forget to draw the state space tree.

4. [10 pt] Give the steps to reduce the clique optimization problem to clique decision problem.
5. [10 pt] Show that the absolute approximation problem corresponding to the clique cover problem is \( \mathcal{NP} \)-hard. The clique cover problem is to find a vertex cover for a clique.