Important: This is an open book test. You can use any books, notes, or paper, but not exchange anything with other students. You are not allowed touse any electronic/communication devices, including a calculator. *Switch off your cell phone or any other electronic communication device*. Any calculations and rough work can be done on the back side of the test pages. You will lose five points for not writing your name.

1. [10 pt] Give the asymptotic upper and lower bounds for T_n in the following recurrences. Assume that T_n is constant for $n \leq 2$. Make your bounds as tight as possible and justify your answers.

   (a) $T_n = 16T_{n/4} + n^2$

   (b) $T_n = T_{\sqrt{n}} + 1$

2. [10 pt] Analyze the computing time of quicksort for average case and worst case.

3. [10 pt] The maximum and minimum numbers in an array of unsorted integers can be found by a linear scan. This will be a $\Theta(n)$ algorithm. Can you perform the operation in $O(\lg n)$ time? Outline your algorithm to do the same.

4. [5 pt] Find an optimal solution to the knapsack instance $n = 7$, $m = 15$, $(p_1, p_2, \ldots, p_7) = (10, 5, 15, 7, 6, 18, 3)$, and $(w_1, w_2, \ldots, w_7) = (2, 3, 5, 7, 1, 4, 1)$.

5. [8 pt] Apply the bottom-up dynamic programming algorithm to the following instance of the 0/1-knapsack problem with a capacity $m = 6$:

$$W = 3, 2, 1, 4, 5$$

$$P = 25, 20, 15, 40, 50$$