

Important: This is an open book test. You can use any books, notes, or paper. *Turn off your cell phone or any other electronic communication device.* Any calculations and rough work can be done on the back side of the test pages. You will lose five points for not writing your name.

1. [10 pt] Prove by induction that $n! = O(n^n)$.

2. [10 pt] Show that

(a) $n^3 + 10^6 n^2 = \Theta(n^3)$

(b) $33n^3 + 4n^2 = \Omega(n^3)$

3. [10 pt] Solve the following recurrence

$$T_n = \begin{cases} 1 & n \leq 4 \\ 2T_{\sqrt{n}} + \log(n) & n > 4 \end{cases}$$

4. [10 pt] Consider the following adjacency matrix representation of an undirected graph. The graph is presented as an upper triangular matrix as the matrix will be symmetric across the principal diagonal. The intersection of row i and column j gives the weight of the edge ij .

	1	2	3	4	5	6	7	8
1	-	11	13		2			
2		-	15	8	12		6	
3			-					
4				-	14		10	17
5					-			5
6						-	21	7
7							-	11
8								-

Find the distance of shortest path from 1 to all other vertices.

5. [10 pt] Solve the assembly line scheduling problem for the following instance:

$$\begin{array}{ll}
 e_1 = 11 & a_{1i} \rightarrow 3 \ 8 \ 2 \ 14 \ 8 \ 2 \\
 e_2 = 17 & a_{2i} \rightarrow 2 \ 4 \ 8 \ 7 \ 7 \ 1 \\
 x_1 = 7 & t_{1,i+1} \rightarrow 4 \ 5 \ 2 \ 6 \ 1 \\
 x_2 = 9 & t_{2,i+1} \rightarrow 4 \ 3 \ 4 \ 1 \ 3
 \end{array}$$