

Important: This is an open book test. You can use any books, notes, or paper. *Do not log into the computer during the test.* Any calculations and rough work can be done on the back side of the test pages. You will lose five points for not writing your name.

1. [5 pt] Prove by induction

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad n \geq 1$$

2. [5 pt] Show that the following equality is correct:

$$5n^2 - 6n = \Theta(n^2)$$

3. [5 pt] Solve the following recurrence relation

$$T_n = \begin{cases} 1 & n \leq 4 \\ T_{\sqrt{n}} + c & n > 4 \end{cases}$$

4. [10 pt] Find an optimal solution to the 0/1 knapsack instance when $n = 7$, $m = 15$, $(p_1, p_2, \dots, p_7) = (10, 5, 15, 7, 6, 18, 3)$, and $(w_1, w_2, \dots, w_7) = (2, 3, 5, 7, 1, 4, 1)$.

5. [10 pt] For the tree in Figure 4.2 in your textbook, solve the TVSP when (a) $\delta = 4$ and (b) $\delta = 6$.