CS 5130 **Advanced Data Structures and Algorithms**

Assignment # 5 Due Date: December 05, 2013

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1. [40 pt] A *bipartite graph* is a graph $G = (V, E)$ whose vertices can be partitioned into two sets ($V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$) such that there are no edges between vertices in the same set (for instance, if $u, v \in V_1$, then there is no edge between u and v). Give a linear-time algorithm to determine whether an undirected graph is bipartite.

2. [40 pt] We have three containers whose sizes are 10pints, 7 pints, and 4 pints, respectively. The 7-pint and 4-pint containers start out full of water, but the 10-pint container is initially empty. We are allowed one type of operation: pouring the contents of one container into another, stopping only when the source container is empty or the destination container is full. We want to know if there is a sequence of pourings that leaves exactly 2 pints in the 7- or 4-pint container.

   (a) Model this as a graph problem: give a precise deifinition of the graph involved and state the specific question about this graph that needs to be answered.

   (b) What algorithm should be applied to solve the problem?

3. [20 pt] Prove that if G is a connected undirected graph with n vertices and $n - 1$ edges, then G is a tree.

4. [25 pt] Develop a nondeterministic algorithm of complexity $O(n)$ to determine whether there is a subset of n numbers a_i, $1 \leq i \leq n$, that sums to m. Assume that the set $a_1, \ldots, a_n$ is given.

5. [25 pt] Show that the knapsack optimization problem reduces to the knapsack decision problem when all the p's, w's, and m are integer and the complexity is measured as a function of input length.

   Hint: If the input length is q, then $\sum p_i \leq n2^q$, where n is the number of objects. Use a binary search to determine the optimal solution value.

What to handin

Handin a hardcopy of all the sources, readme, makefile(s), and results. Create your programs in a directory called *username*.5 where *username* is your login name on admiral. Once you are done with everything, remove the executables and object files, and issue the following commands:

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% cd
% ~bhatias/bin/handin cs5130 5
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