The Deadlock Problem

Law passed by the Kansas Legislature in early 20th century:
“When two trains approach each other at a crossing, both shall come to a full stop and neither shall start upon again until the other has gone.”

Neil Groundwater has the following to say about working with Unix at Bell Labs in 1972:

... the terminals on the development machine were in a common room ... when one wanted to use the line printer. There was no spooling or lockout. `pr myfile > /dev/lp` was how you sent your listing to the printer. If two users sent output to the printer at the same time, their outputs were interspersed. Whoever shouted. “line printer!” first owned the queue.¹

Deadlock or Deadly Embrace

- Permanent blocking of a set of processes that either compete for system resources or communicate with each other
  - Several processes may compete for a finite set of resources
  - Processes request resources and if a resource is not available, enter a wait state
  - Requested resources may be held by other waiting processes
  - Require divine intervention to get out of this problem
- A significant problem in real systems, because there is no efficient solution in the general case
- Deadlock problem is more important because of increasing use of multiprocessing systems (like real-time, life support, vehicle monitoring, multicore utilization, grid processing)
- Important in answering the question about the completion of a process
- Deadlocks can occur with
  - Serially reusable (SR) resources – printer, tape drive, memory
    * A finite set of identical units, with the number of units constant
    * Can be used safely by only one process at a time and are not depleted by that use
    * Units acquired by processes, used, and released later for use by other processes
    * A process may release a unit only if it has previously acquired it
    * Examples include processors, memory, devices, files, databases, and semaphores
  - Consumable resources – messages
    * Resource gets created dynamically and may be destroyed after use
    * Typically no limit on the number of consummable resources of a specific type
    * Examples are messages, signals, interrupts, and information in I/O buffers

Examples of Deadlocks in Computer Systems

- Reusable resources
  - File Sharing
    * Consider two processes $p_1$ and $p_2$
    * They update a file $F$ and require a scratch tape during the updating
    * Only one tape drive $T$ available
    * $T$ and $F$ are serially reusable resources, and can be used only by exclusive access

Resource Management and Deadlocks

* \( p_2 \) needs \( T \) immediately prior to updating

* request operation
  - Blocks the process requesting the resource
  - Puts the process on the wait queue
  - The process is to remain blocked until the requested resource is available
  - If the resource is available, the process is granted exclusive access to it.

* release operation
  - Returns the resource being released to the system
  - Wakes up the process waiting for the resource, if any

* \( p_1 \) and \( p_2 \) may run as follows

\[
\begin{align*}
p_1: & \quad \vdots \quad \vdots \\
    & \text{request}(F) \quad \text{request}(T) \\
    \vdots & \quad \vdots \\
    r_1: & \quad \text{request}(T) \quad \vdots \\
    \vdots & \quad r_2: \quad \text{request}(F) \\
    \vdots & \quad \vdots \\
    \vdots & \quad \text{release}(T) \quad \text{release}(F) \\
    \vdots & \quad \vdots \\
    \vdots & \quad \text{release}(F) \quad \text{release}(T) \\
    \vdots & \quad \vdots \\
    \vdots & \quad \vdots \\
\end{align*}
\]

* \( p_1 \) can block on \( T \) holding \( F \) while \( p_2 \) can block on \( F \) holding \( T \)

- Single Resource Sharing
  * A single SR resource, such as memory \( M \), with \( m \) allocation units shared by \( n \) processes \( p_1, p_2, \ldots, p_n \), \( 2 \leq m \leq n \)
  * Let the sequence of operations by each process be
    \[
    \begin{align*}
    & m_1 = \text{malloc} (1024); \\
    & m_2 = \text{malloc} (1024); \\
    & \ldots \\
    & \text{free} (m_1); \\
    & \text{free} (m_2);
    \end{align*}
    \]
  * Deadlock due to no memory being available and existing processes requesting more memory
  * Fairly common cause of deadlock

* Consumable resources
  - Deadlock with messages
    * A pair of processes \( p_1 \) and \( p_2 \)
    * Each process receives a message from the other process and then, send a message to the other process
      \[
      \begin{align*}
      p_1(): & \quad p_2(): \\
      \vdots & \quad \vdots \\
      \text{receive} (p_2) & \quad \text{receive} (p_1) \\
      \vdots & \quad \vdots \\
      \text{send} (p_2, m_1) & \quad \text{send} (p_1, m_1)
      \end{align*}
      \]
  * Deadlock with blocking receive

* Locking in Database Systems
  - Locking required to preserve the integrity and consistency of databases, with random request patterns
  - Problem when two records to be updated by two different processes are locked
• Deadlocking by nefarious users
  – Given by R. C. Holt
  
  ```c
  void deadlock ( task )
  {
    wait (event);
  } /* deadlock */
  ```

• Effective Deadlocks
  – Milder form of indefinite postponement of processes competing for a resource
  – Exemplified by Shortest Job Next Scheduling

• Deadlocks in Unix
  – Possible deadlock condition that cannot be detected
  – Number of processes limited by the number of available entries in the process table
  – If process table is full, the fork system call fails
  – Process can wait for a random amount of time before forking again
  – Example:
    * 10 processes creating 12 children each
    * 100 entries in the process table
    * Each process has already created 9 children
    * No more space in the process table ⇒ deadlock
  – Deadlocks due to open files, swap space
  – Another cause of deadlock can be due to the inode table becoming full in the filesystem

Deadlocks problem characterization

• Deadlock Detection
  – Process resource graphs

• Deadlock Recovery
  – “Best” ways of recovering from a deadlock

• Deadlock Prevention
  – Not allowing a deadlock to happen

A Systems Model

• Finite number of resources in the system to be distributed among a number of competing processes
• Partition the resources into several classes
• Identical resources assigned to the same class (CPU cycles, memory space, files, tape drives, printers)
• Allocation of any instance of resource from a class will satisfy the request
• State of the OS – allocation status of various resources, and can be changed only by process actions
• Process actions
  – Request a resource
Resource Management and Deadlocks

- Acquire/use a resource
- Release a resource

• Resources acquired and used only through system calls
  - State can be changed by a process only if the process is not blocked
  - New state is any one of a finite number of possibilities

• Allocation record to be maintained as a system table

• Processes to be modeled as nondeterministic entities

• Deadlock when every process is waiting for an event that can be caused by only one of the waiting processes

• Formal model
  1. System \( \langle \sigma, \pi \rangle \)
     - \( \sigma = \{S, T, U, V, \ldots \} \) – system states
     - \( \pi = \{p_1, p_2, \ldots \} \) – processes

  2. Process \( p_i \) – a partial function from system states into nonempty subsets of system states
     \( p_i : \sigma \rightarrow \{\sigma\} \)
     - Process \( p_i \) can change the current system state into one of several possible states, depending on its action
     - \( S \xrightarrow{*} W \) implies
       * \( S = W \)
       * \( S \xrightarrow{p_i} W \) for some \( p_i \)
       * \( S \xrightarrow{p_i} T \) for some \( p_i \) and \( T \), and \( T \xrightarrow{*} W \)

  3. Process \( p_i \) blocked if it cannot change state of the system by any one of its actions (request/acquire/release)
     \[ \not\exists T \mid S \xrightarrow{p_i} T \]

Consider the system \( \langle \sigma, \pi \rangle \) with \( \sigma = \{S, T, U, V\} \) and \( \pi = \{p_1, p_2\} \)
State changes are:
\[
p_1(S) = \{T, U\}, p_1(T) = \Omega, p_1(U) = \{V\}, p_1(V) = \{U\}
\]
\[
p_2(S) = \{U\}, p_2(T) = \{S, V\}, p_2(U) = \Omega, p_2(V) = \Omega
\]

Possible sequence of state changes are: \( S \xrightarrow{1} U, T \xrightarrow{2} V, S \xrightarrow{*} V \)

![Diagram](attachment://deadlock_diagram.png)
4. Process $p_i$ deadlocked in $S$ if
   - $p_i$ is blocked in $S$
   - No operations can make the process to be *unblocked*
   or, we can say that $p_i$ is deadlocked in $S$ if $\forall T \mid S \rightarrow T$, $p_i$ is blocked in $T$

5. Deadlock state $S$ if $\exists p_i$ deadlocked in $S$

6. Safe state $S$ if $\exists T \mid S \rightarrow T$, $T$ is not a deadlock state

**Resource Allocation Graph**

- Directed graph to describe the state of the system of resources and processes
- Set of vertices $V$ consisting of
  - Set of processes $P = P_1, P_2, \ldots$
  - Represent process nodes as circles
  - Set of resource types $R = R_1, R_2, \ldots$
  - Represent resource nodes as squares with a dot (·) representing each instance of the resource
  - Resource types with multiple instances could be I/O devices allocated by a resource management module
- Set of edges $E$
  - Request edge – Directed edge from $P_i$ to $R_j$
    - Denoted by $P_i \rightarrow R_j$
    - $P_i$ has requested for an instance of $R_j$ and is currently waiting for that resource
  - Assignment edge – Directed edge from $R_j$ to $P_i$
    - Denoted by $R_j \rightarrow P_i$
    - An instance of $R_j$ has been allocated to $P_i$
- No cycles in the graph $\Rightarrow$ no deadlock
- Cycle in the graph $\Rightarrow$ deadlock
- Each process involved in a cycle is deadlocked
- Cycle in the resource graph is necessary and sufficient condition for the existence of a deadlock
- If a graph contains several instances of a resource type, a cycle is not a sufficient condition for a deadlock but it still is a necessary condition

**Deadlock Characterization**

- Necessary and sufficient conditions for deadlocks – Four conditions to hold simultaneously
  1. Mutual exclusion
     - Only one process may use a resource at a time
     - At least one resource must be held in a non-sharable mode
  2. Hold and wait
     - Existence of a process holding at least one resource and waiting to acquire additional resources currently held by other processes
  3. No preemption
Resource Management and Deadlocks

4. Circular wait
   - Processes waiting for resources held by other waiting processes

Deadlock Detection

- Do not restrict process actions or limit resource access (if resources are available to satisfy requests)
- Periodically detect the circular wait condition using a deadlock detection algorithm
- Simulate the most favored execution of each unblocked process
  - An unblocked process may acquire all the needed resources
  - Run and then release all the acquired resources
  - Remain dormant thereafter
  - Released resources may wake up some previously blocked process
  - Continue the above steps as long as possible
  - If any blocked processes remain, they are deadlocked

- Reduction of resource graphs
  - Process blocked if it cannot progress by either of the following operations
    * Request
    * Acquisition
    * Release
  - Reduced of resource graph
    * Reduced by a process $p_i$
      - by removing all edges to and from $p_i$
      - $p_i$ is neither blocked nor isolated node
    * After reduction, $p_i$ becomes an isolated node
    * Irreducible if the graph cannot be reduced by any process
    * Completely reducible if a sequence of reductions deletes all the edges in the graph
  - Lemma 1. All reduction sequences of a given resource graph lead to the same irreducible graph.

- Algorithms for deadlock detection with SR resources
  - The Deadlock Theorem. $S$ is a deadlock state if and only if the resource graph of $S$ is not completely reducible.
  - Representation of resource graph
    * Matrix representation – Two $n \times m$ matrices
      1. Allocation matrix $A$ – processes as rows and resources as columns
         $A_{ij}, i = 1, \ldots, n, j = 1, \ldots, m$ gives the number of units of resource $R_j$ allocated to process $p_i$
      2. Request matrix $B$ – Similar to $A$
        $B_{ij}$ gives the number of units of resource $R_j$ requested by process $p_i$
    * Linked list structure – Four lists
      1. Resources allocated to processes
         $p_i \rightarrow (R_x, a_x) \rightarrow (R_y, a_y) \rightarrow \cdots \rightarrow (R_z, a_z)$
      2. Resources requested by processes
      3. Allocation list of processes with respect to a resource
      4. Request list of processes with respect to a resource
Available units vector \((r_1, \ldots, r_m)\)

- Deadlocks detected by looping through the process request lists, making reductions where possible
- Worst case execution time \(mn^2\)
- Algorithm `deadlock`

```c
// Check if the request for process pnum is less than or equal to available vector

bool req_lt_avail ( const int * req, const int * avail, const int pnum, const int num_res )
{
    int i ( 0 );
    for ( ; i < num_res; i++ )
        if ( req[pnum*num_res+i] > avail[i] )
            break;
    return ( i == num_res );
}

bool deadlock ( const int * available, const int m, const int n, const int * request, const int * allocated )
{
    int work[m];       // m resources
    bool finish[n];    // n processes

    for ( int i ( 0 ); i < m; work[i] = available[i++]; )
    for ( int i ( 0 ); i < n; finish[i++] = false );

    int p ( 0 );
    for ( ; p < n; p++ )   // For each process
    {
        if ( finish[p] ) continue;
        if ( req_lt_avail ( request, work, p, m ) )
        {
            finish[p] = true;
            for ( int i ( 0 ); i < m; i++ )
                work[i] += allocated[p*m+i];
            p = -1;
        }
    }

    for ( p = 0; p < n; p++ )
        if ( ! finish[p] )
            break;

    return ( p != n );
}
```

Example
Resource Management and Deadlocks

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Request</th>
<th>Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C</td>
<td>A B C A B C A B C</td>
<td></td>
</tr>
<tr>
<td>$p_0$</td>
<td>0 1 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>$p_1$</td>
<td>2 0 0</td>
<td>2 0 2</td>
</tr>
<tr>
<td>$p_2$</td>
<td>3 0 3</td>
<td>0 0 0</td>
</tr>
<tr>
<td>$p_3$</td>
<td>2 1 1</td>
<td>1 0 0</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0 0 2</td>
<td>0 0 2</td>
</tr>
</tbody>
</table>

No deadlock with the sequence $<p_0, p_2, p_3, p_1, p_4>$

- Consider that $p_2$ makes an additional request for an instance of type $C$

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<td>2 0 2</td>
</tr>
<tr>
<td>$p_2$</td>
<td>3 0 3</td>
<td>0 0 1</td>
</tr>
<tr>
<td>$p_3$</td>
<td>2 1 1</td>
<td>1 0 0</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0 0 2</td>
<td>0 0 2</td>
</tr>
</tbody>
</table>

deadlock with processes $<p_1, p_2, p_3, p_4>$

- **reach(a)** – Set of nodes in the graph reachable from node $a$

- **Theorem 2. The Cycle Theorem.** A cycle in a resource graph is a necessary condition for deadlock.

- **Theorem 3.** If $S$ is not a deadlock state and $S \xrightarrow{\delta} T$, then $T$ is a deadlock state if and only if the operation by $p_i$ is a request and $p_i$ is deadlocked in $T$.

- Special Cases of Resource Graphs
  - **Knot:** A knot in a directed graph $(N, E)$ is a subset of nodes $M \subseteq N$ such that $\forall a \in M$, $\text{reach}(a) = M$
  - Immediate Allocation
    - Expedient States – All processes having requests are blocked
    - Expedient state $\Rightarrow$ A knot in the corresponding resource graph is a sufficient condition for deadlock
  - Single-Unit Resources – Cycle is sufficient and necessary condition for deadlock

**Recovery from Deadlock**

- **Recovery by process termination**
  - Abort all deadlocked processes
    - Most commonly adopted solution
    - Do we really need to abort all the processes or just some of them will do
  - Back up each deadlocked process to some previously defined checkpoint and restart all of them
    - Needs rollback and restart mechanisms built into the system
    - Risk of original deadlock reoccurring but nondeterminacy of concurrent processes may prevent that
  - Terminate deadlocked processes in a systematic way
    - When enough processes terminated to recover from deadlock, stop terminations
    - Perform deadlock detection at each process’ termination
    - Problems with the approach
      - If the process is in the midst of updating a file, its termination may leave the file in an incorrect state
      - If the process is in the midst of printing, the printer must be reset
    - Processes should be terminated based on some criterion/policy

...
Resource Management and Deadlocks

- Priority of a process
- CPU time used and expected usage before completion
- Number and type of resources being used (can they be preempted easily?)
- Number of resources needed for completion
- Number of processes needed to be terminated
- Are the processes interactive or batch?

* Minimum cost recovery based on
  - Cost of destroying a process
  - Cost of recovery from the next process state

- Recovery by resource preemption
  - Enough resources to be preempted from processes and made available to deadlocked processes to resolve the deadlock
  - Selecting a victim
  - Rollback
  - Prevention of starvation – Ensure that the resources are not always preempted from the same process

Deadlock Prevention

- Uses a conservative resource allocation policy; undercommits resources
- Each process can request and acquire all the needed resources at the same time
  - Works well for processes that perform a single burst of activity
  - No preemption necessary
  - Grossly inefficient
  - May delay process initialization
  - Processes must identify all future resource requirements in advance

- Deny one of the required conditions for a deadlock
  - Mutual Exclusion
    * Cannot be done for non-sharable resources (like printers)
    * Sharable resources (read-only files) do not require mutually exclusive access ⇒ cannot be involved in deadlock
    * Cannot deny mutual exclusion as some resources are inherently non-sharable
  - Hold and Wait
    * Processes can request and acquire all the resources at one time
    * Request resources only if the process is holding none
      - If the process is holding any resources, they must be released before requests can be granted
    * Disadvantages
      - Low resource utilization – resources may get allocated but not used for a long time
      - Possibility of starvation – on popular resources
  - No Preemption
    - If a process holding resources requests for another resource that cannot be immediately allocated, all currently held resources are preempted
    - Process restarted only when it regains all the resources
    - Suitable for resources whose state can be easily saved – CPU registers, memory
  - Circular Wait
Resource Management and Deadlocks

- Impose a total ordering on all resource types
- Each process requests resources in an increasing order of enumeration
- If several instances of a resource required, a single request must be issued for all of them

Deadlock Prevention based on Maximum Claims

- Also called Deadlock Avoidance
- A priori knowledge of maximum possible claims for each process
- Dynamically examine the resource allocation status to ensure that no circular wait condition can exist
- Resource allocation state
  - Defined by the number of available and allocated resources, and the maximum demands of the processes
  - Safe, if the system can allocate resources to each process (up to its maximum) in some order and still avoid a deadlock
- System in safe state only if there exists a safe sequence
- All unsafe states are not deadlock states
- An unsafe state may lead to a deadlock
- Example

<table>
<thead>
<tr>
<th>Process</th>
<th>Max needs</th>
<th>Allocation</th>
<th>Current needs</th>
</tr>
</thead>
<tbody>
<tr>
<td>p₀</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>p₁</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>p₂</td>
<td>9</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

Current availability: 3

Safe sequence: \( \langle p₁, p₀, p₂ \rangle \)

- Possible to go from a safe state to an unsafe state

- Let the state after allocating two tapes to process \( p₁ \) be

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<td>0</td>
</tr>
<tr>
<td>p₂</td>
<td>9</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

Current availability: 1

Let \( p₂ \) request and acquire the last remaining tape drive
- Mistake in allocating one more tape drive to \( p₂ \)
- Problem: To detect the possibility of unsafe state and deny requests even if resources are still available
- Banker’s Algorithm
  - Based on banking system that never allocates its available cash such that it can no longer satisfy the needs of all its customers

- Deadlock Avoidance
  - Requires a process to declare the maximum instances of each resource type needed
  - Upon request, the system must determine whether the allocation will leave the system in a safe state
  - Number of processes in the system \(- n\)
  - Number of resource classes \(- m\)
Data structures
* available
  - A vector of length \( m \)
  - Number of available resources of each type
  - \( \text{available}[j] = k \Rightarrow k \) instances of resource class \( R_j \) are available
* maximum
  - An \( n \times m \) matrix
  - Defines maximum demand for each process
  - \( \text{maximum}[i,j] = k \Rightarrow \) process \( p_i \) may request at most \( k \) instances of resource class \( R_j \)
* allocation
  - An \( n \times m \) matrix
  - Defines the number of resources of each type currently allocated to each process
  - \( \text{allocation}[i,j] = k \Rightarrow \) process \( p_i \) is currently allocated \( k \) instances of resource class \( R_j \)
* need
  - An \( n \times m \) matrix
  - Indicates the remaining resource need of each process
  - \( \text{need}[i,j] = k \Rightarrow \) process \( p_i \) may need \( k \) more instances of resource type \( R_j \) in order to complete its task
  - \( \text{need}[i,j] = \text{maximum}[i,j] - \text{allocation}[i,j] \)

Banker’s Algorithm
* request
  - Request vector for process \( p_i \)
  - \( \text{request}[i,j] = k \Rightarrow \) process \( p_i \) wants \( k \) instances of resource class \( R_j \)
* Upon request for resources, the following actions are taken
  \[
  \text{if ( request}[i] > \text{need}[i] )} \\
  \text{throw ( "Asked for more than initial max request" );}
  \]
  \[
  \text{if ( request}[i] <= \text{available })} \\
  \{ \\
  \text{available }= \text{request}[i]; \\
  \text{allocation}[i] += \text{request}[i]; \\
  \text{need}[i] -= \text{request}[i]; \\
  \}
  \text{else} \}
  \text{p}[i].\text{wait(); } // \text{Put process in wait state}
* If resulting resource-allocation state is safe, transaction is completed and process \( p_i \) is allocated its resources
* If the new state is unsafe, \( p_i \) must wait for \( \text{request}[i] \) and the old allocation state is restored

Safety Algorithm
* Hypothetically allocates the desired resources to processes
* Finds out whether or not the system is in a safe state using the deadlock algorithm

Example
* System with five processes

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Maximum</th>
<th>Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>( p_4 )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Matrix need

```
<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>p0</td>
<td>7</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>p1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>p2</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>p3</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>p4</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
```

Sequence \( \langle p_1, p_3, p_4, p_2, p_0 \rangle \) satisfies the safety criterion

Let process \( p_1 \) request one additional instance of resource class A and two additional instances of resource class C

\[ \text{request}_1 = (1, 0, 2) \]

\( \text{request}_1 \leq \text{available} \) is true

New state

```
<table>
<thead>
<tr>
<th></th>
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<th>C</th>
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</tbody>
</table>
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Sequence \( \langle p_1, p_3, p_4, p_0, p_2 \rangle \) satisfies the safety criterion

Request for \( (3, 3, 0) \) by \( p_4 \) cannot be granted