CS 313	**Analysis of Algorithms**	Test 3
Name:	Summer 1999	Max Pts: 55

Important: This is an open book test. You can use any books, notes, or paper. Any calculations and rough work can be done on the back side of the test pages. You will loose five points for not writing your name.

1. [5 pt] What is the maximum and minimum numbers of elements in a heap of height h? Is an array that is in reverse sorted order a heap? Is the sequence

   $$\langle 23, 17, 14, 6, 13, 10, 1, 5, 7, 12 \rangle$$

   a heap?

2. [10 pt] Show the operation of `heap_extract_max` on the heap

   $$A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$$

3. [10 pt] Give the average-case and best-case recurrence for quicksort. Show that the best case running time from quicksort is $\Omega(n \lg n)$. You must solve the recurrence formally.

4. [10 pt] Consider the following version of `counting_sort`

```
counting_sort(A,B,k)
   for i ← k do
      c[i] ← 0
   for j ← 1 to length[A] do
      C[A[j]] ← C[A[j]] + 1
   for i ← 2 to k do
      C[i] ← C[i] + C[i-1]
   for j ← 1 to length[A] do
      B[C[A[j]]] ← A[j]
      C[A[j]] ← C[A[j]] - 1
```

Does the algorithm still work properly? If it does, comment on its stability. If it does not, explain why. What will it take to fix it?

5. [10 pt] Demonstrate the insertion of the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \bmod 9$. Also show the insertion of values using *open addressing* with *linear probing* using the same hash function.

6. [10 pt] Draw the top-down 2-3-4 tree built when the keys

$$E \quad A \quad S \quad Y \quad Q \quad U \quad E \quad S \quad T \quad I \quad O \quad N$$

are inserted (in that order) into an initially empty tree. Also give the equivalent red-black tree.

7. [5 pt; Bonus] Suppose that we have numbers between 1 and 1000 in a binary search tree and want to search for the number 363. Which of the following sequences could *not* be the sequence of nodes examined? Explain the reason for your answer.

   (a) 2, 252, 401, 398, 330, 344, 397, 363

   (b) 924, 220, 911, 244, 898, 258, 362, 363

   (c) 925, 202, 911, 240, 912, 245, 363

   (d) 2, 399, 387, 219, 266, 382, 381, 278, 363

   (e) 935, 278, 347, 621, 299, 392, 358, 363