CS 278	**Design and Analysis of Algorithms**	Test 1
Name:	Winter 2001	Max Pts: 49

Important: This is an open book test. You can use any books, notes, or paper. Any calculations and rough work can be done on the back side of the test pages. You will lose five points for not writing your name.

1. [6 pt] Show the contents of the array after each `union` operation when you use the quick-find algorithm to solve the connectivity problem for the sequence 5-6, 2-4, 2-6, 4-1, 4-6, 6-1, and 3-1. Also give the number of times the program accesses the array for each input pair. Will the result be any different if we reverse the order of some pairs?

2. [8 pt] Consider the problem of linear search. How many elements of the input sequence need to be checked on the average, assuming that the element being searched for is equally likely to be any element in the array. How about in the worst case? What are the average case and worst case running times of linear search in Θ-notation?

3. [5 pt] What is the smallest value of n such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine?

4. [10 pt] Give asymptotically tight bounds on the following summation:

$$\sum_{k=1}^{n} k^r$$

5. [10 pt] Use iterative method to show that the solution of

$$T_n = T_{\lceil \frac{n}{2} \rceil} + 1$$

is $O(\lg n)$.

6. [10 pt] Use master method to give tight asymptotic bounds for the following recurrence:

$$T_n = 4T_{\frac{n}{2}} + n$$