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**Note:** Create a subdirectory in your home directory and call it `your_last_name.5`, where `your_last_name` is your real last name (for example, `bhatia` for me). Do all the programs for this assignment in that directory. After you are done, submit the code by typing the following command:

```
~sanjiv/bin/handin your_last_name.5 cs278 5
```

Again, do not forget to substitute for `your_last_name`. The command should be executed from your home directory.

### 1. Rotating a vector

You are required to rotate an  $n$ -element vector  $x$  left by  $i$  positions in time proportional to  $n$ , and with as little extra space as possible. For example, the vector of characters

VLOMGQXVKURGLTLISJRV

when rotated left by  $i = 4$ , results in

GQXVKURGLTLISJRVLOM

One might try to solve the problem by copying the first  $i$  elements of  $x$  into a temporary array, moving the remaining  $n - i$  elements left  $i$  places, and then copying the first  $i$  from the temporary array back to the last positions in  $x$ . However, the  $i$  extra locations used by this scheme make it too space-expensive. For a different approach, we could define a function to rotate  $x$  left by one position (in time proportional to  $n$ ) and call it  $i$  times, but that would be too time-expensive.

To solve the problem within the resource bounds will apparently require a more complicated program. One successful approach is a delicate juggling act: move  $x[0]$  to the temporary  $t$ , then move  $x[i]$  to  $x[0]$ ,  $x[2i]$  to  $x[i]$ , and so on (taking all indices into  $x$  modulo  $n$ ), until we come back to taking an element from  $x[0]$ , at which point we instead take the element from  $t$  and stop the process.

If that process didn't move all the elements, then we start at  $x[1]$  and continue until we move all the elements.

Write a program to implement the vector rotation as described. Make sure that it works with all the possible values of  $i$  and  $n$ . You may have to think in terms of the greatest common divisor of  $i$  and  $n$  for some help.

### 2. Swapping two segments of a vector

This is a different view of the above problem: rotating the vector  $x$  is really just swapping the two segments of the vector  $ab$  to be the vector  $ba$ , where  $a$  represents the first  $i$  elements of  $x$ . Suppose  $a$  is shorter than  $b$ . Divide  $b$  into  $b_l$  and  $b_r$  so that  $b_r$  is the same length as  $a$ . Swap  $a$  and  $b_r$  to transform  $ab_lb_r$  into  $b_rb_la$ . The sequence  $a$  is in its final place, so we can focus on swapping the two parts of  $b$ . Since this new problem has the same form as the original, we can solve it recursively.

### 3. Swapping two segments of a vector (again)

Let us view the problem as transforming the array  $ab$  into array  $ba$ , but let us also assume that we have a function that reverses the elements in a specified portion of the array. Starting with  $ab$ , we reverse  $a$  to get  $a^r b$ , reverse  $b$  to get  $a^r b^r$ , and then reverse the whole thing to get  $(a^r b^r)^r$ , which is exactly  $ba$ . This results in the following code for rotation; the comments show the results when input string  $x$  is rotated left four elements.

```
x = "VLOMGQXVKU"
i = 4
n = strlen ( x );
reverse ( 0, i-1 )           /* MOLVGQXVKU */
reverse ( i, n-1 )          /* MOLVUKVXQG */
reverse ( 0, n-1 )          /* GQXVKUVLOM */
```

Write a program to achieve the vector rotation using this algorithm.